Received: *November 11, 1999,* Accepted: *November 18, 1999*

Universality of $1/Q$ corrections revisited

Mrinal Dasgupta

Dipartimento di Fisica, Università di Milano Bicocca, and
INFN, Sezione di Milano
Via Celoria 16, I-20133, Milano, Italy
E-mail: dasgupta@mi.infn.it

Lorenzo Magnea

Dipartimento di Fisica Teorica, Università di Torino, and
INFN, Sezione di Torino
Via P. Giuria 1, I-10125, Torino, Italy
E-mail: magnea@to.infn.it

Graham E. Smye*

Cavendish Laboratory, University of Cambridge
Madingley Road, Cambridge CB3 OHE, U.K.
E-mail: smye@hep.phy.cam.ac.uk

Abstract: We provide an exact analytical calculation at the two-loop level in the abelian limit of the leading power correction for the C parameter in e^+e^- annihilation. We compare our results to the numerical value obtained employing the soft approximation, the abelian part of the Milan factor. We demonstrate that a simple proportionality holds between the leading power corrections to the C parameter and to the longitudinal cross section in the soft region, and we verify that this proportionality holds for the full two-loop abelian contribution computed here. We comment on the possibility of extending this technique to other event shape variables and distributions, as well as to the non-abelian contributions.

Keywords: LEP HERA and SLC Physics, QCD, NLO Computations, Jets.

*Research supported by the U.K. Particle Physics and Astronomy Research Council.

Contents

1. Introduction

The study of event shape variables in both e^+e^- annihilation and DIS is at present one of the most active areas of phenomenological investigation in QCD. From a theoretical viewpoint existing predictions for the distributions of these variables emerge from the application of several of the most advanced techniques of perturbative QCD, hence it is particularly satisfying to be able to confront theory successfully with experiment. Theoretical predictions for power corrections to shape variable distributions in fact require all order resummations of perturbation theory, either within the framework of soft gluon resummation [1, 2, 3, 4, 5], or in the context of renormalon techniques [6, 7, 8, 9, 10], extended to account for the necessary two-loop effects [11, 12, 13, 14]. In either case, the theoretical framework is adjusted for the inclusion of non-perturbative power corrections, using methods and models that may ultimately pave the way for a better understanding of low energy behaviour of QCD [10, 5]. On the experimental side, these variables have proved relatively simple to measure and have been one of the most popular sources for the extraction of the strong coupling from fits to the data [15, 16, 17, 18].

In the present paper we will make use of the dispersive approach developed in [6, 7, 8, 9, 10]. In its original form, the dispersive approach was based on the resummation of fermionic bubble chains, a procedure which is strictly consistent only in the abelian limit, but can be reformulated under rather general assumptions in terms of an effective low-energy QCD coupling, for which a dispersive representation is adopted. Since the dispersive variable plays a role which is formally similar to a gluon mass, this approach is sometimes referred to as the "massive gluon scheme". It was soon realized (first in a study by Nason and Seymour [19], who considered the thrust variable) that this method could not directly be applied to general event shape variables, which are sensitive to the details of the decay process of the particular gluon being treated as "massive". A method to deal with the problem was proposed in refs. [20, 11], where it was shown that the bubble resummation could be performed without integrating inclusively over the gluon decay products, and a calculation of the leading power corrections to the longitudinal cross section was performed in the abelian limit. A much more general analysis of the effects of non-inclusiveness on the most commonly used event shape variables was performed in refs. [12, 13], where the two-loop correction due to gluon splitting was computed including non-abelian terms. The main result of this detailed analysis is that the whole effect of non-inclusiveness, within the framework of the dispersive approach, is largely universal for the shape variables studied. Each observable acquires a two-loop enhancement factor to the naive "massive gluon" calculation, and this enhancement factor, now called the Milan factor, is observable independent. All dependence on the chosen observable is encoded in the massive gluon result, so that the phenomenology is basically unaffected. Moreover, shape variables in DIS receive an identical enhancement [14].

The calculations leading to the Milan factor are done using soft two-loop matrix elements. This is justified because shape variables, being by construction essentially linear in particle momenta, are expected to receive leading ($1/Q$) power corrections only from soft gluon emission. In addition, soft kinematics is employed, where effects such as terms involving the square of the small transverse momentum of the gluon decay products are discarded, since they can only contribute at the level of $1/Q^2$ corrections. The universality of soft gluon radiation coupled with an underlying geometrical universality (linearity in emitted transverse momenta) in the shape variables themselves thus leads to the universal Milan factor appearing in every case.

In the present paper we test some of these assumptions by considering the $1/Q$ correction to two event shape variables, the C parameter and the longitudinal cross section. In the case of the C parameter we perform the full calculation in the abelian (large n_f) limit, in a manner analogous to the longitudinal cross section calculation of [11], and we arrive at an analytical result for the $1/Q$ behaviour. The correction to the C parameter was calculated in the soft approximation in ref. [13], and the Milan factor enhancement obtained for it. The results we get here should be directly comparable to the abelian part of the Milan factor, since we allow only for gluon splitting into quarks.

We also show that a simple relationship holds between the C parameter and the longitudinal cross section in the soft approximation. This relation is respected by the $1/Q$ corrections computed here for C and in reference [11] for the longitudinal cross section, suggesting that the $1/Q$ corrections should be correctly obtained from the simplified soft approximation advocated in the computation of the Milan factor.

We find however that the numerical result for the two-loop enhancement factor for the C-parameter in the abelian limit disagrees with the abelian limit of the Milan factor: if we take the ratio of our result to the massive gluon one, we find an enhancement factor of $15\pi^2/128 = 1.157$, whereas the corresponding enhancement factor computed numerically in ref. [13] is of the form $1 + r_{ni}^{(a)} = 1.078$, where $r_{ni}^{(a)}$ is the "non-inclusive" piece of the Milan factor in the abelian limit.[1] In essence, the two results would agree if the correction factor $r_{ni}^{(a)}$ were doubled.

The layout of the present paper is as follows. In section 2 we introduce the observables C and σ_L, and we demonstrate their equivalence in the soft limit. In section 3 we show our calculation of the C parameter in some detail. In the final section we compare our results to those obtained by other authors and briefly discuss the different methods, as well as possible developments and applications.

2. Definitions and the soft approximation

2.1 C parameter

The C parameter [21] can be defined as

$$C = 3 - \frac{3}{2}\sum_{i,j=1}^{n} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)(p_j \cdot q)}\,, \tag{2.1}$$

where q^μ is the photon four-momentum, $q^2 = Q^2$, and the sum is over all outgoing particles, so that each pair of particles is counted twice. An equivalent definition, valid in the e^+e^- centre of mass frame, is

$$C = \frac{3}{2}\sum_{i,j=1}^{n} \frac{|\vec{p_i}||\vec{p_j}|}{Q^2}\sin^2\theta_{ij}\,, \tag{2.2}$$

where the sum once again runs over all outgoing particles. From these definitions, it is clear that the C parameter vanishes at the Born level. The first contribution within perturbation theory appears at $\mathcal{O}(\alpha_s)$ when one has a three body final state.

2.2 Longitudinal cross section

The longitudinal cross section can be defined, within the context of single particle inclusive annihilation, by considering the differential cross section for the production

[1] To arrive at this number for $r_{ni}^{(a)}$ we take $C_A = 0$ in ref. [12, eq. (4.16)], in the numerator as well as in β_0.

JHEP11(1999)025

of a hadron with given energy and angle with respect to the beam axis. The angular dependence of this cross section can be organized in the form [22]

$$\frac{d^2\sigma^h}{dx d\cos\theta} = \frac{3}{8}\left(1+\cos^2\theta\right)\frac{d\sigma_T^h}{dx} + \frac{3}{4}\sin^2\theta\frac{d\sigma_L^h}{dx} + \frac{3}{4}\cos\theta\frac{d\sigma_A^h}{dx}\,, \tag{2.3}$$

with $x = 2p_h \cdot q/q^2$ being the hadron energy fraction and θ its angle with respect to the beam direction. The three terms on the right-hand side are referred to as the transverse, longitudinal and asymmetric contributions. The first two contributions respectively arise from gauge boson polarisation states transverse and longitudinal to the direction of the outgoing hadron h; the last term is a parity violating contribution that is absent in purely electromagnetic annihilation.

The longitudinal cross section itself is usually defined as the first moment of $d\sigma_L^h/dx$, and it can be projected out from the single particle inclusive differential cross section in eq. (2.3) by multiplying with a suitable weight function and integrating over $\cos\theta$ and x, according to

$$\sigma_L = \frac{1}{2}\sum_h \int_0^1 x dx \int_{-1}^1 d\cos\theta\,(2-5\cos^2\theta)\frac{d^2\sigma^h}{dx d\cos\theta}\,. \tag{2.4}$$

Note that since we are measuring an inclusive cross-section we can replace the hadronic sum above by the corresponding partonic one.

In the next subsection we shall make use of the above definitions to write down expressions for the longitudinal cross section and the C parameter in the soft approximation, relevant to the computation of $1/Q$ corrections.

2.3 Soft approximation

At the partonic level, in the soft approximation, the annihilation process produces a pair of hard fermions ("primary" quark and antiquark), dressed by soft gluon radiation. The primary quark and antiquark are essentially back to back and have an angular distribution about the beam axis that may be approximated by the purely transverse $1+\cos^2\theta_0$ pattern (the first term in eq. (2.3)), where θ_0 is the quark angle with respect to the beam. In this approximation one is neglecting the recoil of the hard fermions due to soft gluon emission.

Let us consider the contribution to σ_L of a soft gluon, which is known to be the term responsible for the appearance of a $1/Q$ correction [11, 23]. In order to do this, we need to introduce the gluon angle with respect to the beam θ_g, while the gluon direction with respect to the parent quark may be specified by a polar angle θ and an azimuthal angle ϕ.[2] In terms of these variables, the soft gluon contribution to the

[2]An identical calculation relevant to the flux tube model of hadronisation was described by Nason and Webber [24].

longitudinal cross section can be written as

$$\sigma_L^g = \frac{1}{2}\int_0^1 dx x \int_{-1}^1 d\cos\theta_g\, d\cos\theta_0\, d\cos\theta \int_0^{2\pi}\frac{d\phi}{2\pi}\left(2-5\cos^2\theta_g\right)\times$$
$$\times\frac{3}{8}(1+\cos^2\theta_0)\frac{d^2\sigma^g}{dxd\cos\theta}\delta\left(\cos\theta_g-\cos\theta_0\cos\theta+\sin\theta_0\sin\theta\cos\phi\right)$$
$$=\frac{1}{4}\int_0^1 dx x\int_{-1}^1 d\cos\theta\sin^2\theta\frac{d^2\sigma^g}{dxd\cos\theta}\,, \tag{2.5}$$

where x is the gluon energy fraction, $x = 2E_g/Q$, while here $d^2\sigma^g/dxd\cos\theta$ is the differential cross section for soft gluon emission, given in the soft approximation by the standard $q\bar{q}$ antenna pattern.

The relationship with the C parameter is easily established by noting that the only terms in the sum over outgoing particles that will contribute to the $1/Q$ correction for the mean value of C are the terms pairing one of the primary fermions with the soft gluon. The contribution from the pairing of the primary quark to the antiquark is $\mathcal{O}(k_\perp^2)$, so that it can be discarded in the back-to-back (soft) limit. One finds then that in the soft approximation, concentrating on the terms responsible for $1/Q$ corrections, the mean value of C can be calculated by weighting the matrix element $d^2\sigma^g/dxd\cos\theta$ with the function

$$C^{(1/Q)} = \frac{6}{Q^2}\frac{Q}{2}E_g\sin^2\theta = \frac{3}{2}x\sin^2\theta\,, \tag{2.6}$$

which is precisely six times the weighting function for σ_L as given in eq. (2.5). One concludes that the power corrections to the expectation value of C and to σ_L are proportional, according to

$$\langle C\rangle^{(1/Q)} = 6\,\sigma_L^{(1/Q)}\,. \tag{2.7}$$

In the next section we will calculate the exact value of the power correction for the C parameter, taking proper account of gluon decay but in a large n_f limit. We will then be able to compare with the calculation of [11], and verify that the $1/Q$ corrections to $\langle C\rangle$ and σ_L indeed obey eq. (2.7).

3. Calculations

In this section we provide some details of our calculation of the average $\langle C\rangle$ due to gluon splitting into a ("secondary") quark-antiquark pair. We will begin by performing the calculation in the naive massive gluon scheme, although such a calculation is manifestly incomplete, since we shall need it in order to compare our results with other available calculations.

JHEP11(1999)025

3.1 Massive gluon scheme

In the dispersive approach (for a review, see [25]), for a sufficiently inclusive observable, defined as an observable insensitive to the momentum distribution of the gluon decay products, it is possible to classify power corrections by computing the observable at one loop with Feynman rules appropriate to a massive gluon. One can then show [6, 7, 8, 9, 10] that power corrections are in one-to-one correspondence with nonanalytic terms in the expansion of the result in powers of the squared gluon mass. It is easy to see that the C parameter, like most event shape variables, is sensitive to the details of gluon decay, and thus does not belong to the class of observables that can be dealt with using this method. One can however use the massive gluon result as a first estimate of the size of the correction, and parametrize the full result as a (perturbative) enhancement of the massive gluon calculation. This is particularly useful since one can argue [12, 13] that all dependence on the particular observable at hand is contained in the massive gluon result.

Let us then consider a $q, \bar{q}, g$ final state and let us label the four-momenta of the quark, antiquark and gluon by p_1, p_2 and k respectively. The C parameter assumes the simple form

$$C = 6\frac{(1-x_1)(1-x_2)(1-x_3)}{x_1 x_2 x_3}\,, \tag{3.1}$$

where $x_{1,2} = 2p_{1,2}\cdot q/q^2$ are the energy fractions carried by the quark and the antiquark respectively, while $x_3 = 2k\cdot q/q^2$ is the gluon energy fraction. Energy conservation implies $\sum_i x_i = 2$. Notice that although we are doing a massive gluon calculation we have discarded the gluon mass in the definition of the event shape. This approximation has no effect on the coefficient of the $1/Q$ correction, in the present case; moreover, in order to compare our results with the Milan factor, we need to adopt the massless definition of C in accordance with the procedure of refs. [12, 13].

The expectation value of C in the present approximation is given by

$$\begin{aligned}\langle C\rangle_{(mg)} &= \frac{1}{\sigma_0}\int d\sigma\, C \\ &= \frac{\alpha_s}{16\pi N_c}\int_0^{1-\epsilon} dx_1 \int_{1-x_1-\epsilon}^{1-\frac{\epsilon}{1-x_1}} dx_2\, W^{\mu\alpha}W^*_{\mu\alpha}\, C\,,\end{aligned} \tag{3.2}$$

where σ_0 is the Born cross section, ϵ is the square of the gluon mass divided by q^2, and $(-ie)(-ig)W^{\mu\alpha}$ is the matrix element for the decay of the virtual photon (with polarisation index μ) into a quark, an antiquark, and a gluon with polarisation index α. Current conservation then implies that $q_\mu W^{\mu\alpha} = 0$ and $k_\alpha W^{\mu\alpha} = 0$. In terms of the energy fractions x_i,

$$W^{\mu\alpha}W^*_{\mu\alpha} = 8N_c C_F\left[\frac{x_1^2 + x_2^2 + 2\epsilon(x_1+x_2+\epsilon)}{(1-x_1)(1-x_2)} - \frac{\epsilon}{(1-x_1)^2} - \frac{\epsilon}{(1-x_2)^2}\right]. \tag{3.3}$$

The integrals in eq. (3.2) are easily computed. Expanding the result in powers of ϵ we get

$$\langle C \rangle_{(mg)} = \frac{C_F}{2\pi}\alpha_s \left[4\pi^2 - 33 - 12\pi\sqrt{\epsilon} + \mathcal{O}(\epsilon)\right]. \tag{3.4}$$

In the language of the dispersive approach, this translates into a power correction, generated by the $\sqrt{\epsilon}$ term, given by (see [10])

$$\langle C \rangle^{1/Q}_{(mg)} = 6\pi \frac{A_1}{Q}, \tag{3.5}$$

where the dimensional parameter A_1 is interpreted as the first (half integer) moment of the non-perturbative component of the effective coupling, as defined in [10],

$$A_1 = \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu\,\delta\alpha_{\rm eff}(\mu^2). \tag{3.6}$$

A completely equivalent expression for the power correction follows from a purely perturbative analysis, as a consequence of the sum over "bubble" graphs (see [6, 7]). In that case the coefficient A_1 is related to a slightly different effective coupling function, related to the present one essentially by a change of renormalization scheme (for a comparison of the two points of view, see [25]).

3.2 C parameter with gluon splitting

Let us now consider the splitting of the gluon into a quark-antiquark pair in more detail. We focus on a four-particle final state with a "primary" $q\bar{q}$ pair, carrying momenta p_1 and p_2, while the secondary quark and antiquark have momenta k_1 and k_2. Summing the relevant graphs, and summing in each graph over all insertions of fermion bubbles in the gluon propagator, we are led to replace eq. (3.2) by

$$\begin{aligned}\langle C \rangle = {} & \frac{3}{N_c}\left(\frac{2\pi}{Q^2}\right)^2 \alpha_s^2 T_R n_f \int \frac{dk^2}{k^4}\frac{1}{|1+\Pi(k^2)|^2}\int d\,\mathrm{Lips}[q\to p_1,p_2,k]\, W_{\mu\alpha}\, W^*_{\nu\beta}\, L^{\mu\nu} \times \\ & \times \int d\,\mathrm{Lips}[k\to k_1,k_2]\,\mathrm{Tr}[\gamma^\alpha \not{k}_1 \gamma^\beta \not{k}_2]\, C\,. \end{aligned} \tag{3.7}$$

Here $d\,\mathrm{Lips}[q \to \{p_i\}]$ is the appropriate Lorentz-invariant phase space measure, $\Pi(k^2)$ is the renormalized one-loop gluon vacuum polarization induced by quarks, and $L^{\mu\nu}$ is the leptonic tensor, which will be substituted by the corresponding average over the beam orientation, $\langle L^{\mu\nu}\rangle = -4Q^2 g^{\mu\nu}/3$, as customary when working with event shape variables. In eq. (3.7) we have factorized the four-particle phase space to introduce an explicit integral over the square of the gluon four-momentum, k^2, which of course plays the role of the dispersive variable ("gluon mass") in the present calculation. Thus we will be interested in the k^2-dependent integrand of eq. (3.7), and in particular in its expansion in powers of $\sqrt{k^2}$. Notice that a factor of $1/(1+\Pi(k^2))$ is just what is needed to turn the (fixed) coupling α_s into the running coupling evaluated at scale k^2, in the abelian limit. For C we take the full expression for 4 outgoing particles, which we write as

$$C = C_{(p)} + C_{(m)} + C_{(s)} \,, \tag{3.8}$$

where $C_{(p)}$ is the "inclusive" term, involving only the momenta of the primary fermions,

$$C_{(p)} = 3 - 3\frac{(p_1 \cdot p_2)^2}{(p_1 \cdot q)(p_2 \cdot q)} \,, \tag{3.9}$$

$C_{(s)}$ is the term involving only the momenta of the secondary fermions,

$$C_{(s)} = -3\frac{(k_1 \cdot k_2)^2}{(k_1 \cdot q)(k_2 \cdot q)} \,, \tag{3.10}$$

and finally $C_{(m)}$ is the sum of all mixed terms. Using the symmetries of the integral in eq. (3.7) we can freely replace k_2 with k_1 and p_2 with p_1 in $C_{(m)}$, thus we can use

$$C_{(m)} = -12\frac{(p_1 \cdot k_1)^2}{(p_1 \cdot q)(k_1 \cdot q)} \,. \tag{3.11}$$

Since we are interested in the distribution of C as a function of the gluon "mass" k^2, for each of the three contributions to $\langle C \rangle$ we write

$$\langle C \rangle_{(i)} = \int \frac{d\epsilon}{\epsilon} \frac{1}{\left|1 + \Pi(Q^2 \epsilon)\right|^2} \mathcal{C}_{(i)}(\epsilon) \,, \tag{3.12}$$

where $i = p, m, s$ and $\epsilon = k^2/Q^2$. Our task is to compute the distributions $\mathcal{C}_{(i)}(\epsilon)$, and extract the non-analytic behaviour for small values of ϵ.

The primary contribution to C is clearly the easiest to evaluate, since the integration over k_1 and k_2 can be done inclusively. Using the transversality of the hadronic tensor, $k_\alpha W^{\mu\alpha} = 0$, one can simply substitute for the bubble integral

$$\int d\,\mathrm{Lips}[k \to k_1, k_2]\, \mathrm{Tr}[\gamma_\alpha \not{k}_1 \gamma_\beta \not{k}_2] \to -\frac{1}{6\pi}\, Q^2\, \epsilon\, g_{\alpha\beta} \,. \tag{3.13}$$

One finds then

$$\mathcal{C}_{(p)}(\epsilon) = \frac{1}{N_c} \frac{\alpha_s^2 T_R n_f}{48\pi^2} \int_0^{1-\epsilon} dx_1 \int_{1-x_1-\epsilon}^{1-\frac{\epsilon}{1-x_1}} dx_2\, W^{\mu\alpha} W^*_{\mu\alpha}\, C_{(p)} \,. \tag{3.14}$$

The remaining integrals are easily performed, and expanding around $\epsilon = 0$ yields

$$\mathcal{C}_{(p)}(\epsilon) = \frac{C_F}{2\pi} \frac{\alpha_s^2 T_R n_f}{3\pi} \left[-8 \ln \epsilon - \frac{133}{6} + \mathcal{O}(\epsilon)\right] . \tag{3.15}$$

Notice that $\mathcal{C}_{(p)}(\epsilon)$ by itself is infrared divergent. The divergence will be cancelled by the mixed contribution, to which we now turn.

The ϵ-distribution arising from the mixed terms is by far the most difficult to evaluate analytically. It is given by

$$\mathcal{C}_{(m)}(\epsilon) = \frac{3}{N_c}\frac{\alpha_s^2 T_R n_f}{2\pi}\frac{1}{Q^2\epsilon}\int_0^{1-\epsilon} dx_1 \int_{1-x_1-\epsilon}^{1-\frac{\epsilon}{1-x_1}} dx_2 \, \frac{1}{p_1\cdot q} W^{\mu\alpha}W_{\mu}^{*\beta}T_{\alpha\beta}\,, \tag{3.16}$$

where

$$T_{\alpha\beta} = \int d\,\mathrm{Lips}[k\to k_1,k_2]\,\mathrm{Tr}[\gamma_\alpha \not{k}_1\gamma_\beta \not{k}_2]\frac{(p_1\cdot k_1)^2}{k_1\cdot q}\,. \tag{3.17}$$

There are several ways to evaluate the bubble integral $T_{\alpha\beta}$. Perhaps the most straightforward is to note that $T_{\alpha\beta}$ is symmetric, and obeys $k^\alpha T_{\alpha\beta}=0$. Then we can write the decomposition

$$\begin{aligned} T_{\alpha\beta} = {} & A_1\,Q^2\left(g_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2}\right) + A_2\left(p_{1\alpha}-\frac{p_1\cdot k}{k^2}k_\alpha\right)\left(p_{1\beta}-\frac{p_1\cdot k}{k^2}k_\beta\right) + \\ & + A_3\left[\left(p_{1\alpha}-\frac{p_1\cdot k}{k^2}k_\alpha\right)\left(q_\beta-\frac{q\cdot k}{k^2}k_\beta\right) + (\alpha\leftrightarrow\beta)\right] + \\ & + A_4\left(q_\alpha-\frac{q\cdot k}{k^2}k_\alpha\right)\left(q_\beta-\frac{q\cdot k}{k^2}k_\beta\right), \end{aligned} \tag{3.18}$$

where the A_i are scalar functions of p_1, k and q, which can be evaluated, for example, by integrating eq. (3.17) component by component in the rest frame of k.

It follows from eqs. (3.16) and (3.18), and from current conservation, that only four independent projections of the tensor $W^{\mu\alpha}W_\mu^{*\beta}$ are needed. The projection with the metric tensor $g_{\alpha\beta}$ gives eq. (3.3); the other projections are given by

$$\begin{aligned} p_{1\alpha}p_{1\beta}W^{\mu\alpha}W_\mu^{*\beta} &= 4N_cC_FQ^2\frac{x_1^2(x_3-1-\epsilon)}{(1-x_1)^2} \\ (p_{1\alpha}q_\beta+q_\alpha p_{1\beta})W^{\mu\alpha}W_\mu^{*\beta} &= 8N_cC_FQ^2\left[\frac{x_1^2(x_3-1-\epsilon)}{(1-x_1)^2}+\frac{(x_3-1-\epsilon)^2}{(1-x_1)(1-x_2)}\right] \\ q_\alpha q_\beta W^{\mu\alpha}W_\mu^{*\beta} &= 4N_cC_FQ^2\left[\frac{x_1^2(x_3-1-\epsilon)}{(1-x_1)^2}+\frac{2(x_3-1-\epsilon)^2}{(1-x_1)(1-x_2)}+\right. \\ &\qquad\left. + \frac{x_2^2(x_3-1-\epsilon)}{(1-x_2)^2}\right]. \end{aligned} \tag{3.19}$$

To simplify the computation, a convenient change of variables is

$$\begin{aligned} u &= x_1 - x_2 \\ v+\frac{\epsilon}{v} &= 2-x_1-x_2 = x_3\,. \end{aligned} \tag{3.20}$$

In terms of these variables, the scalars entering the decomposition of the tensor $T_{\alpha\beta}$, eq. (3.18), have the general form

$$\frac{Q^2}{p_1\cdot q}A_i = \frac{P_i(u,v,\epsilon)+Q_i(u,v,\epsilon)\ln v + R_i(u,v,\epsilon)\ln\epsilon}{(v^2-\epsilon)^9(v^2-2v-uv+\epsilon)}\,, \tag{3.21}$$

where P_i, Q_i and R_i are polynomial functions of u, v and ϵ, whereas the projections of the hadronic tensor given in eq. (3.19) are rational functions of the same variables. All the integrals involved in the computation of eq. (3.16) can be performed analytically, through a rather lengthy process of partial fractioning. The result is expressible in terms of di- and trilogarithms of functions of ϵ, and can be expanded around $\epsilon = 0$ yielding

$$\mathcal{C}_{(m)}(\epsilon) = \frac{C_F}{2\pi}\frac{\alpha_s^2 T_R n_f}{3\pi}\left[8\ln\epsilon + 4\pi^2 - \frac{65}{6} - \frac{45\pi^3}{32}\sqrt{\epsilon} + \mathcal{O}(\epsilon)\right]. \tag{3.22}$$

As promised, the infrared divergence (as $\epsilon \to 0$) of eq. (3.22) cancels the one of eq. (3.15).

The final contribution to $\langle C\rangle$ is that from the secondary terms, arising from the distribution $\mathcal{C}_{(s)}(\epsilon)$. By inspection, this contribution must be at least $\mathcal{O}(\epsilon)$, since the corresponding event shape, eq. (3.10) is quadratic in the gluon energy in the soft region, and thus cannot contribute to the leading power correction. This can be confirmed by an explicit calculation along the lines of the ones leading to eqs. (3.15) and (3.22).

Our final result for the C parameter ϵ distribution is

$$\mathcal{C}(\epsilon) = \frac{C_F}{2\pi}\frac{\alpha_s^2 T_R n_f}{3\pi}\left[4\pi^2 - 33 - \frac{45\pi^3}{32}\sqrt{\epsilon} + \mathcal{O}(\epsilon)\right]. \tag{3.23}$$

Following [6, 7, 11], one can directly compare eq. (3.23) with eq. (3.4), upon noticing that the coefficient $\alpha_s^2 T_R n_f/(3\pi)$ should be read as $\alpha_s^2\beta_0^f$, where β_0^f is the fermion contribution to the one-loop β function. From here the power correction can be obtained by using the renormalisation group equation for the running coupling to replace the "spectral function", $\beta_0^f\alpha_s^2(k^2)$, by the logarithmic derivative of the coupling with respect to its argument and integrating by parts. The coefficient of $\sqrt{\epsilon}$ is mapped to the coefficient of the $1/Q$ correction as in eqs. (3.4) and (3.5).

Using the language of refs. [12, 13], one would interpret eq. (3.23) as the sum of the "naive" contribution to the series of power corrections plus the fermionic part of the "non inclusive" correction to it. We find that the exact calculation leading to eq. (3.23) gives a $1/Q$ correction larger than the one computed in the massive gluon scheme by a factor $15\pi^2/128 = 1.157$, as announced in the introduction.

3.3 Comparison with σ_L

A calculation analogous to the one just outlined was performed for the longitudinal cross section in ref. [11], where a model for the distribution in energy fraction of the power correction was also constructed. It was also noted there that the $1/Q$ correction to σ_L arises from a resummation of power corrections of the form $1/(xQ)^{2n}$ in the distributions, which have no correction with odd powers of $1/Q$; this was also the conclusion reached in [23], within the massive gluon model. This shows that

JHEP11(1999)025

the leading power correction to the total σ_L is entirely due to the soft gluon region, $x \to 0$. The result given in ref. [11] for σ_L can be written in the form

$$\sigma_L(\epsilon) = \frac{C_F}{2\pi} \frac{\alpha_s^2 T_R n_f}{3\pi} \left[1 - \frac{15\pi^3}{64} \sqrt{\epsilon} + \mathcal{O}(\epsilon) \right] . \tag{3.24}$$

One sees that the two results indeed satisfy eq. (2.7), which was obtained in the soft approximation. This confirms that the leading power correction arises entirely from the emission of soft gluons, and from their subsequent splitting.

4. Discussion

We performed an analytical calculation, with two-loop accuracy and in the abelian limit, of the leading power correction to the C parameter measured in e^+e^- annihilation; we showed that our result is simply related to the corresponding result for the longitudinal cross section, and we compared it to existing calculations. We explicitly checked that the entire power correction comes from the region in which the splitting gluon is soft: this can be shown by tracking the contributions to the final answer through the calculation, and is confirmed by the fact that the simple relationship we find between $\langle C \rangle$ and σ_L is a property of the soft approximation. Our calculation is thus a positive test of the applicability of the soft approximation to the computation of $1/Q$ corrections to event shape observables. We find however a numerical discrepancy with the results of [12, 13], which we should reproduce in the abelian limit.

Our technique, in essence a straightforward if lengthy two-loop calculation, is applicable in principle to all event shape variables. Although in some cases it may turn out to be too cumbersome to generate the full analytical result, it is always possible, and in fact simple, to produce a one-dimensional integral representation for the answer, from which the coefficient of the desired power correction can always be derived numerically. The fact that the calculation is performed in the abelian limit is not a severe problem, it should rather be viewed as a technical simplification.

Corrections to the abelian limit come from two sources: on the one hand, a subset of the non-abelian diagrams serves to reconstruct the full one-loop β function, β_0, from its abelian counterpart, β_0^f; on the other hand, the fact that the event shape is sensitive to the details of gluon splitting generates contribution that are not proportional to β_0, since they are not directly related to the running of the coupling. The first source of corrections (which must be taken into account also in the simplifed massive gluon scheme) is under control: there are strong physical arguments for such corrections to be there, and furthermore, at least in principle, they could be explicitly included by using existing techniques to isolate gauge-invariant subsets of diagrams such as the pinch technique [26]. The second set of corrections is related to the splitting of the gluon into two gluons, and these are the corrections included in the soft approximation in the Milan factor. If it were to prove impossible to iron out the numerical differences between the two approaches, an analytic computation of non-abelian gluon splitting can in principle be performed.

A more interesting issue from a phenomenological point of view is the study of event shape distributions, such as $d\sigma/dC$ or $d\sigma_L/dx$, with x the energy fraction of the detected hadron. Such studies can be performed with the technique outlined here, and in fact in [11] the σ_L distribution was studied, showing how the $1/Q$ correction to σ_L arises from a summation of even power correction to all orders. However, having shown that the power correction to the average event shape is determined by soft gluon emission, we expect to recover at least qualitatively the results of [12, 13], namely the characteristic constant shift in the distribution from its perturbative estimate by a power suppressed amount. It would be interesting to understand to what extent this simple behaviour of the distributions is due to the approximations inherent in the dispersive approach. In fact the shift in the distributions is recovered in a factorization-based approach [5] only as an approximation of the full answer, obtained essentially by neglecting the long-range, wide-angle correlations of the soft radiation. A complete description of the power correction to the distribution requires in that approach a non-perturbative function rather than a single parameter. The phenomenological impact of a more detailed analysis of the power correction along these lines is at present an open question.

Acknowledgments

We would like to thank Yuri Dokshitzer, Giuseppe Marchesini, Gavin Salam, Bryan Webber and Joannis Papavassiliou for helpful discussions and clarifications. One of us (M.D.) would like to acknowledge the hospitality of the HEP group at the Cavendish Laboratory in Cambridge and of the INFN Sezione di Torino where part of this work was carried out. The work was supported in part by the EU Fourth Framework Programme 'Training and Mobility of Researchers', Network 'Quantum Chromodynamics and the Deep Structure of Elementary Particles', contract FMRX-CT98-0194 (DG 12 - MIHT).

Note. After the discrepancy between our results and those of refs. [12, 13] had become evident, an error of a factor of two was found in the computation of the Milan factor [27]. Once this error is corrected, there is complete agreement with our calculation. Moreover it becomes apparent that σ_L indeed belongs to the family of observables that receive the universal Milan enhancement, there no longer being any conflict with ref. [11]. This should enable experimental investigation of the power correction to σ_L just as for other shape variables.

References

[1] S. Catani, L. Trentadue, G. Turnock and B.R. Webber, *Thrust distribution in e^+e^- annihilation*, *Phys. Lett.* **B 263** (1991) 491.

[2] S. Catani, G. Turnock and B.R. Webber, *Jet broadening measures in e^+e^- annihilation*, *Phys. Lett.* **B 295** (1992) 269.

[3] S. Catani and B.R. Webber, *Resummed C parameter distribution in e^+e^- annihilation*, *Phys. Lett.* **B 427** (1998) 377 [`hep-ph/9801350`].

[4] G.P. Korchemsky and G. Sterman, *Universality of infrared renormalons in hadronic cross-sections*, `hep-ph/9505391`.

[5] G.P. Korchemsky and G. Sterman, *Power corrections to event shapes and factorization*, *Nucl. Phys.* **B 555** (1999) 335 [`hep-ph/9902341`].

[6] M. Beneke and V.M. Braun, *Naive nonabelianization and resummation of fermion bubble chains*, *Phys. Lett.* **B 348** (1995) 513 [`hep-ph/9411229`].

[7] P. Ball, M. Beneke and V.M. Braun, *Resummation of $(\beta_0\alpha_s)^n$ corrections in QCD: techniques and applications to the τ hadronic width and the heavy quark pole mass*, *Nucl. Phys.* **B 452** (1995) 563 [`hep-ph/9502300`].

[8] Y.L. Dokshitzer and B.R. Webber, *Calculation of power corrections to hadronic event shapes*, *Phys. Lett.* **B 352** (1995) 451 [`hep-ph/9504219`].

[9] R. Akhoury and V.I. Zakharov, *On the universality of the leading, $1/Q$ power corrections in QCD*, *Phys. Lett.* **B 357** (1995) 646 [`hep-ph/9504248`]; *Leading power corrections in QCD: from renormalons to phenomenology*, *Nucl. Phys.* **B 465** (1996) 295 [`hep-ph/9507253`].

[10] Y.L. Dokshitzer, G. Marchesini and B.R. Webber, *Dispersive approach to power behaved contributions in QCD hard processes*, *Nucl. Phys.* **B 469** (1996) 93 [`hep-ph/9512336`].

[11] M. Beneke, V.M. Braun and L. Magnea, *Phenomenology of power corrections in fragmentation processes in e^+e^- annihilation*, *Nucl. Phys.* **B 497** (1997) 297 [`hep-ph/9701309`].

[12] Y.L. Dokshitzer, A. Lucenti, G. Marchesini and G.P. Salam, *Universality of $1/Q$ corrections to jet shape observables rescued*, *Nucl. Phys.* **B 511** (1998) 396 [`hep-ph/9707532`].

[13] Y.L. Dokshitzer, A. Lucenti, G. Marchesini and G.P. Salam, *On the universality of the Milan factor for $1/Q$ power corrections to jet shapes*, *J. High Energy Phys.* **05** (1998) 003 [`hep-ph/9802381`].

JHEP11(1999)025

[14] M. Dasgupta and B.R. Webber, *Two loop enhancement factor for $1/Q$ corrections to event shapes in deep inelastic scattering*, *J. High Energy Phys.* **10** (1998) 001 [hep-ph/9809247].

[15] J.C. Thompson, *Measurements of alpha$_s$ from hadronic event shapes in e^+e^- annihilation*, hep-ex/9812004.

[16] DELPHI collaboration, *Energy dependence of event shapes and of α_s at LEP2*, *Phys. Lett.* **B 456** (1999) 322.

[17] P.A.M. Fernandez, O. Biebel and S. Bethke, *Tests of power corrections to event shape distributions from e^+e^- annihilation*, hep-ex/9906033.

[18] H1 collaboration, *Event shapes and power corrections in ep DIS*, hep-ex/9906002.

[19] P. Nason and M.H. Seymour, *Infrared renormalons and power suppressed effects in e^+e^- jet events*, *Nucl. Phys.* **B 454** (1995) 291 [hep-ph/9506317].

[20] M. Beneke, V.M. Braun and L. Magnea, *On the infrared sensitivity of the longitudinal cross-section in e^+e^- annihilation*, *Nucl. Phys.* **54A** *(Proc. Suppl.)* (1997) 183 [hep-ph/9609266].

[21] R.K. Ellis, D.A. Ross and A.E. Terrano, *The perturbative calculation of jet structure in e^+e^- annihilation*, *Nucl. Phys.* **B 178** (1981) 421.

[22] B. Mele and P. Nason, *The fragmentation function for heavy quarks in QCD*, *Nucl. Phys.* **B 361** (1991) 626.

[23] M. Dasgupta and B.R. Webber, *Power corrections and renormalons in e^+e^- fragmentation functions*, *Nucl. Phys.* **B 484** (1997) 247 [hep-ph/9608394].

[24] P. Nason and B.R. Webber, *Scaling violation in e^+e^- fragmentation functions: QCD evolution, hadronization and heavy quark mass effects*, *Nucl. Phys.* **B 421** (1994) 473, erratum *ibid.* **480** (1996) 755.

[25] M. Beneke, *Renormalons*, *Phys. Rep.* **317** (1999) 1 [hep-ph/9807443].

[26] J. Papavassiliou, *The pinch technique approach to the physics of unstable particles*, hep-ph/9905328.

[27] B.R. Webber, private communication;
see also Yu.L. Dokshitzer *Perturbative QCD and power corrections*, to appear in the Proceedings of the 11th Rencontres de Blois, *Frontiers of matter*, 1999, hep-ph/9911299.